

UNCLASSIFIED

AD 268 222

*Reproduced
by the*

**ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA**



UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

A S T I A 2 2 2 2 2 2 2

FEASIBILITY STUDY AND EXPERIMENTAL MODEL OF AN OPTIMAL CONTROL
SERVOMECHANISM.

Report No. 1

Contract SC 87256

1st Quarterly Report

DA 36-039

First Quarterly Progress Report, 15 June to 15 September 1961.

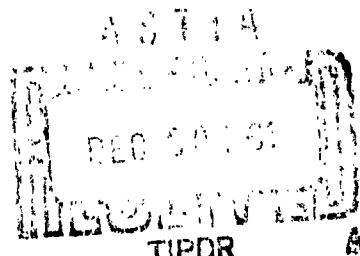
U.S. Army Signal Research and Development Laboratory
Fort Monmouth, New Jersey

Code
164-50

Aeronca Manufacturing Corporation
Aerospace Division
P.O. Box 536
Baltimore, Maryland

62-1-5

XEROX



ASTIA Availability Notice:

Qualified requestors may obtain copies of this report from ASTIA.

**Feasibility Study and Experimental Model of an Optimal Control
Servomechanism.**

Report No. 1
Contract No. SC 87256
DA 36-039

First Quarterly Progress Report, 15 June to 15 September 61.

Object of Research: To develop an optimal servo-motor.

By Wm. S. Hare

TABLE OF CONTENTS

	Pg
1.0 Purpose	1
2.0 Abstract	3
3.0 Conferences and Publications	
3.1 Conferences	5
3.2 Publications	5
4.0 Factual Data	
4.1 Theoretical Work	6
4.1.1 List of Symbols	13
5.0 Program for Next Interval and Conclusions	
5.1 Practical Implementation	15
5.2 Problem Areas	15
5.3 Proposed Solutions	16
5.4 Present Status	17
5.5 Summary	18
6.0 Identification of Personnel	20
7.0 Abstract Card	21
Fig. 1 Pictorial of Motor & Load Configuration	6a
Fig. 2 Torque Speed Curves	7
Fig. 3 Phase-plane Semi-trajectories	9a
Fig. 4 Second Order Switching Boundary	10a
Fig. 5 Phase-plane Trajectories	11a
Fig. 6 Carthesian Maps of Ambiguous Phase-plane . . .	11b
Fig. 7 Truncated Phase-plane Plot	11c
Fig. 8 Cylindrical Map of Phase-plane	11d
Fig. 9 Carthesian Plot of Phase-plane Showing High . .	12a
Gear Ratio	
Fig. 10 Cylindrical Plot of Preceding Page	12b

1.0 PURPOSE

The purpose of this project shall be the construction of a feasibility model demonstrating the principles of optimal discontinuous control for servo motors. The particular servomechanism to be constructed will be optimally designed with respect to inputs of the type, step plus ramp disturbance, and will be self adaptive to torque variations in the load. There appear to be two possible design approaches to the synthesis of the optimal switching function. One of these would be to construct an analog of the switching function using conventional analog computer techniques, such as logarithmic amplifiers, multipliers, dividers and other nonlinear elements to produce, in real time, subject to the state variations of the error and error rate in the phase-plane, an optimal discontinuous forcing function.

A second synthesis technique would be to construct the switching function faster than real time through the use of passive analog elements. This prediction technique would run the linear plant to the origin, in the phase-plane, many times faster than real time. Noting the error and switching the forcing function will match that predicted condition. This second technique has been adopted by the project managers of this contract for the following reasons:

1. It appears to be economically much more feasible than the fabrication of many expensive nonlinear computing elements.
2. It is expected that the accuracy of the passive elements involved will be higher than those of the equivalent analog synthesis techniques.
3. The technique appears to be new and therefore represents

a step forward in the state of the control system's art with respect to relay actuated control systems.

4. It is not felt that this technique is in any real sense a deviation from the statement of work of the contract in the sense that the optimal switching curve is indeed fabricated, faster than real time, and sampling techniques provide us with the answer as to whether or not the linear plant is following the optimal switching function.

While on the face of it there appears to be a loss of generality in constructing a faster than real time analog for the second order servomechanism, it is also true that in any generalized n-th order control system's problem, the use of logarithmic amplifiers in analog synthesis is mandatory. Once the system has been reduced to the n-1 order, in phase-space, the final solution could be mechanized in the same manner that it is being produced in this particular project. A completely general approach would be to use the logarithmic amplifier synthesis technique for the entire control system's problem. However, due to the unavailability of sufficiently accurate log amplifiers, it is felt that a feasibility determination can be made on the basis of this technique. Further studies will improve the techniques required to mechanize the completely general n-th order problem.

2.0 ABSTRACT

Using results obtained by Dr. R.W. Bass, who introduced the concept of relay feedback into the field of control theory, it has been possible to mathematically describe a phase plane portrait of the switching characteristic of an extremal torque, time optimal, positional-servomechanism which is self adaptive to torque load variations.

This problem has been of historical interest to workers in the field, and, at the present time, of accentuated practical interest in the field of guidance and control. Because of this new emphasis on extremal and optimal control the current feasibility effort was instituted. The simplifying assumptions and mathematical idealizations have been dropped and an engineering attack has been mounted on the practical, the realizable, the economic, and the uncertain factors.

The approach has been twofold; theoretical work which removes the original simplifications, introduces uncertain parameters and considers the inherent difficulties of extremal systems as immediate problem objectives; practical implementation, as the second major effort, has been devoted to consideration of the factors influencing the choice of design, generation of system block diagrams (both functional and operational) and the construction of feasibility hardware to justify our choice of system elements.

Both of these efforts have produced problem areas and compromise structures. In many cases the choice of solution is either arbitrary or must be based on inadequate engineering data or theoretical research which has not yet been accomplished. In view of these prob-

lem areas it is felt that several meetings with Mr. Carl Allen, the Contracting Officer's Designated Representative, would be advisable to resolve these issues.

3.0 CONFERENCES AND PUBLICATIONS

3.1 Conferences

Dr. R.W. Bass, principal investigator and W.S. Hare, project manager, attended the Joint Automatic Control Conference, 1961, with Carl Allen of the Signal Corps and had several important informal discussions with leaders in the field of optimal and adaptive control on subjects relating closely to this project.

Dr. Bass and Mr. Hare met on 16 October 1961 with Samuel Adler, Carl Allen and Peter Devreoces of the Signal Corps to discuss the status of the project and plan the future work.

3.2 Publications

No publications have been approached to date, however there appears to be a class of unusual problems represented in the multipli-connected phase-space difficulty which would justify publication in advanced journals.

4.0 FACTUAL DATA

4.1 Theoretical Work

The theoretical work has been devoted to mathematical effort to obtain a rigorous switching characteristic in closed form which at the same time includes consideration of the fact that any desired final position of a rotating servomechanism may be taken modulo 2π . This means that we have a multipli connected phase space and this is a new consideration in this field.

In addition, computer simulation studies have been instituted, primarily as a means of establishing practical tolerances on the system hardware to attain an acceptable limit cycle, stability margins and the effect of stochastic variations on the input signal and other system parameters.

In our effort to derive a switching characteristic applicable to practical transducers in a medium power servomechanism, the following calculations have been made.

Consider the following configuration for motor and load, in which the following idealizations have been assumed:

- a. Motor transfer function is linear and second order.
- b. Gear train is free of backlash.
- c. Load parameters are constant.
- d. External torque variations never exceed the stall torque of the motor.

See Figure 1.

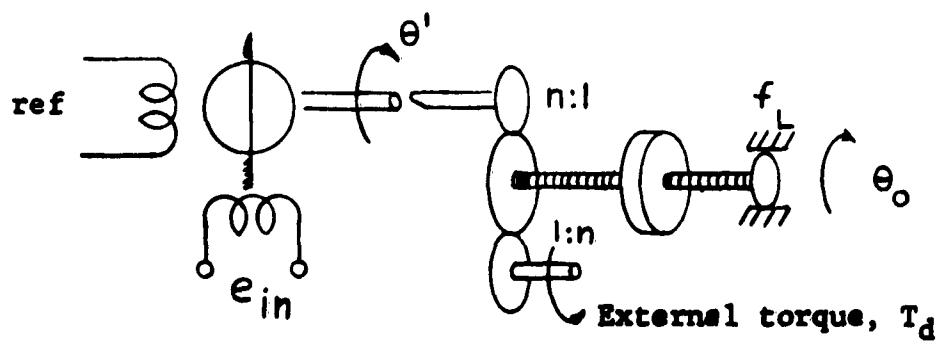


Figure 1

Pictorial of Motor and Load Configuration

If the servo torque-speed curves have the following form:

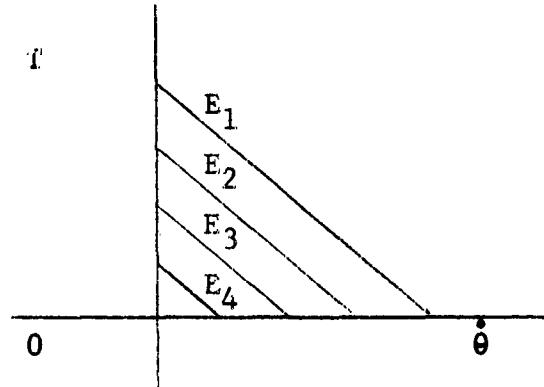


Figure 2.

and $E_1 > E_2 > E_3 \dots$

and since

$$T = \frac{\partial T}{\partial E} A + \frac{\partial T}{\partial \theta} \dot{\theta}$$

then let

$$\frac{\partial T}{\partial E} = K_e; \quad \frac{\partial T}{\partial \theta} = -K_n$$

Hence the developed torque

$$(1) \quad T_m = K_e E_{in} - K_n \dot{\theta}$$

The load torque at the motor shaft

$$(2) \quad T_L = \left(\frac{J_L}{n^2} + J_m \right) \ddot{\theta} + \left(\frac{f_L}{n^2} + f_m \right) \dot{\theta} - T_d$$

Equating (1) and (2) we have the system differential equations

$$(3) \quad \left(\frac{J_L}{n^2} + J_m \right) \ddot{\theta} + \left(\frac{f_L}{n^2} + f_m + K_n \right) \dot{\theta} = K_e E_{in} + T_d$$

Let

$$\left(\frac{J_L}{n^2} + J_m \right) = J, \quad \frac{T_d}{n} = T_d$$

$$\left(\frac{f_L}{n^2} + f_m + K_n \right) = f$$

$$\theta' = n\theta$$

$$\frac{K_e E_{in}}{n} = F \operatorname{sgn} \sigma \quad \{ \text{the extremal constraint} \}$$

Hence (3) becomes

$$(4) \quad J \ddot{\theta} + f \dot{\theta} = F \operatorname{sgn} \sigma + T_d$$

If the solution of this equation is to follow a given input, $\theta_i(t)$ so chosen that

$$(4a) \quad \theta_i(t) = a + bt, \quad |b| < \frac{F+T_d}{f} \quad (\text{runaway velocity})$$

within any finite interval, then it is convenient to define a new variable, $e(t)$, the actuating error

$$(5) \quad e(t) = (\theta_i(t) - \theta(t))$$

clearly $|e(t)| \rightarrow 0$ as $\theta(t) \rightarrow \theta_i(t)$.

After substitution of (5) and (4a), (4) becomes

$$(6) \quad J \ddot{e} + f \dot{e} = (fb - T_d) - F \operatorname{sgn} \sigma$$

We then desire that choice of σ which will bring $e(t)$ and its derivative to the origin from any initial position (e_0, \dot{e}_0) .

Let us solve (6)

Set

$$e(0) = e_0$$

$$\dot{e}(0) = \dot{e}_0$$

Now

$$e(t) = A + B e^{-\frac{f}{J}t} + \frac{1}{f} \left[(f_b - T_d) - F \operatorname{sgn} \sigma \right] t$$

$$\dot{e}(t) = -\frac{f}{J} B e^{-\frac{f}{J}t} + \frac{1}{f} \left[(f_b - T_d) - F \operatorname{sgn} \sigma \right]$$

then

$$e_0 = A + B$$

$$\dot{e}_0 = -\frac{f}{J} B + \frac{1}{f} \left[(f_b - T_d) - F \operatorname{sgn} \sigma \right]$$

hence

$$B = \frac{J}{f^2} \left[(f_b - T_d) - F \operatorname{sgn} \sigma \right] - \frac{J}{f} \dot{e}_0$$

$$A = e_0 + \frac{J}{f} \dot{e}_0 - \frac{J}{f^2} \left[(f_b - T_d) - F \operatorname{sgn} \sigma \right]$$

Finally

$$(7) \quad e(t) = e_0 + \frac{J}{f} \dot{e}_0 - \frac{J}{f^2} \left[(f_b - T_d) - F \operatorname{sgn} \sigma \right] + \left\{ \frac{J}{f^2} \left[(f_b - T_d) - F \operatorname{sgn} \sigma \right] - \frac{J}{f} \dot{e}_0 \right\} e^{-\frac{f}{J}t} + \frac{1}{f} \left[(f_b - T_d) - F \operatorname{sgn} \sigma \right] t$$

and

$$(8) \quad \dot{e}(t) = \frac{1}{f} \left[(f_b - T_d) - F \operatorname{sgn} \sigma \right] - \frac{1}{f} \left[(f_b - T_d) - F \operatorname{sgn} \sigma - f \dot{e}_0 \right] e^{-\frac{f}{J}t}$$

If we eliminate the parameter, t , the resultant equation defines two families of phase plane trajectories, where the phase plane is defined to be $e(t)$, $\dot{e}(t)$ plane. These two families differ in the assumed algebraic sign σ . See the following sketches wherein it is assumed that b , the input rate, and T_d , the disturbing torque, are held constant throughout the trajectory

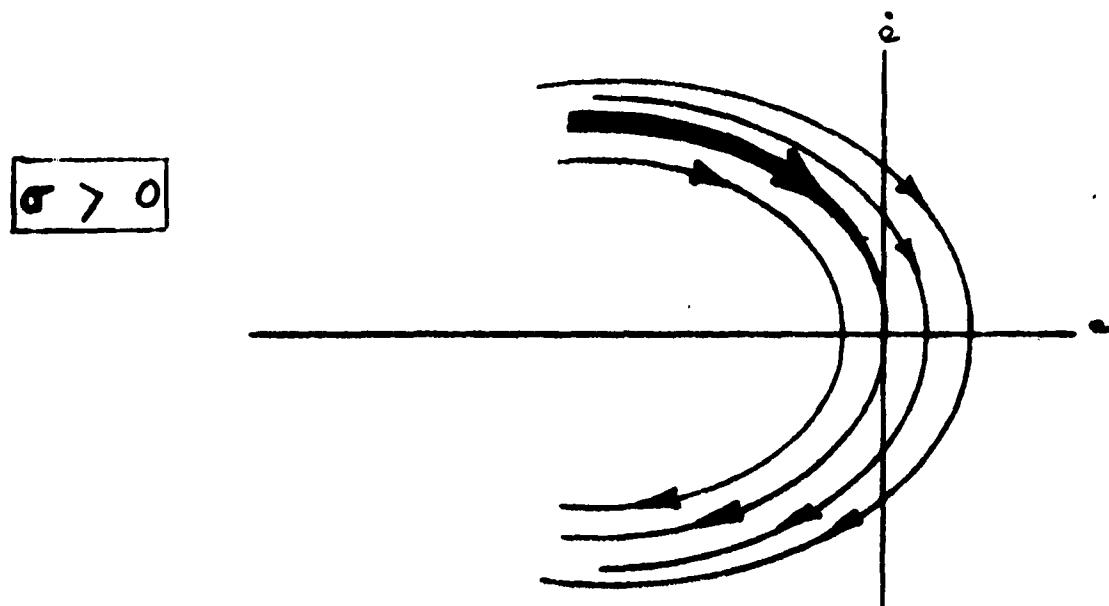
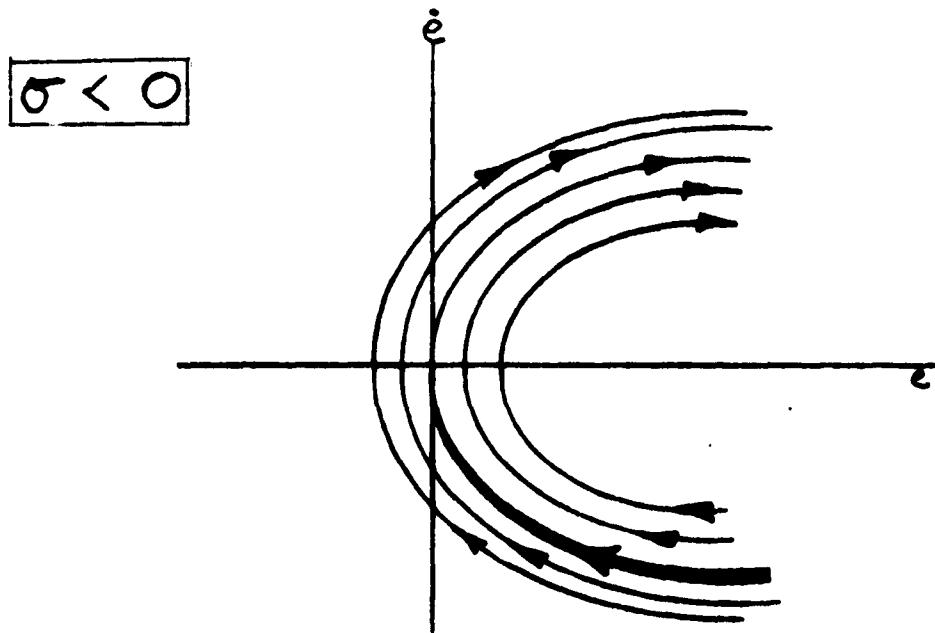


Figure 3.
Phase-plane Semi-trajectories

Note that only that part of the trajectory which is heavily shaded in the sketch, will bring the system to the origin with no torque reversal. This then is the desired final trajectory. Let us solve equations (7) and (8) for this trajectory by eliminating t , and setting the endpoint equal to the origin, $e(t) = \dot{e}(t) = 0$. Hence, (8) becomes

$$(9) \quad e^{-\frac{f}{J}t} = \frac{\frac{1}{f}[(fb - T_d) - F \operatorname{sgn} \sigma]}{\frac{1}{f}[(fb - T_d) - F \operatorname{sgn} \sigma] - \dot{e}_o}$$

and

$$(9a) \quad t = \frac{J}{f} \log \left\{ \frac{\frac{1}{f}[(fb - T_d) - F \operatorname{sgn} \sigma] - \dot{e}_o}{\frac{1}{f}[(fb - T_d) - F \operatorname{sgn} \sigma]} \right\}$$

Clearing (7) and (8) we have

$$(10) \quad 0 = e_o + \frac{J}{f} \dot{e}_o + \frac{J}{f^2} [(fb - T_d) - F \operatorname{sgn} \sigma] \log \left\{ \frac{[(fb - T_d) - F \operatorname{sgn} \sigma] - f \dot{e}_o}{[(fb - T_d) - F \operatorname{sgn} \sigma]} \right\}$$

Equation (10) is a double valued implicit function of (e_o, \dot{e}_o, σ) .

See illustration on page 10a.

Note that in the I and III quadrant solutions, it is implied by equation (9a) that we reach the origin in negative time, i.e., an unrealizable solution.

In other words, we require that equation (10) holds in reality only if

$$(11) \quad \operatorname{sgn} \sigma = \operatorname{sgn} e$$

Hence

$$(12) \quad 0 = e_o + \frac{J}{f} \dot{e}_o - \frac{J}{f^2} [(T_d - fb) + F \operatorname{sgn} \dot{e}_o] \log \left\{ 1 + \frac{f \dot{e}_o}{[(T_d - fb) + F \operatorname{sgn} \dot{e}_o]} \right\}$$

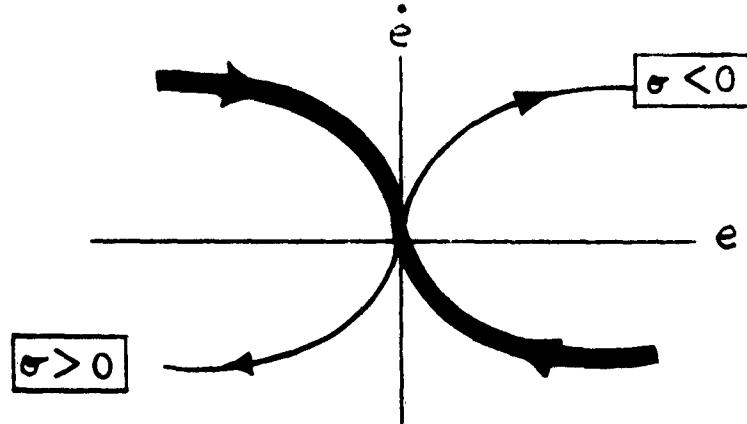


Figure 4.

Second Order Switching Boundary

becomes the optimal trajectory, and indeed it will be shown, is the switching curve.

If in the derivation of (12) we let the end point be given

$$e(t) = e_1$$

$$\dot{e}(t) = 0$$

then

$$(12a) \quad e_1 = e_0 + \frac{J \dot{e}_0}{f} - \frac{J}{f^2} \left[(T_d - f b) + F \operatorname{sgn} \dot{e}_0 \right] \log \left\{ 1 + \frac{f \dot{e}_0}{[(T_d - f b) + F \operatorname{sgn} \dot{e}_0]} \right\}$$

which implies that the system comes to rest with an overshoot of e_1 .

Note that the sign of e_1 is also the sign of the applied torque which will cause the system to evolve to a point of interception with the ideal trajectory (12).

Hence, if we set $e_1 = 0$ and the applied torque equal to, $F \operatorname{sgn} \sigma$, then we generate the complete closed loop ideal switching characteristic for arbitrary initial conditions.

$$(13) \quad \sigma = e_0 + \frac{J \dot{e}_0}{f} - \frac{J}{f^2} \left[(T_d - f b) + F \operatorname{sgn} \dot{e}_0 \right] \log \left\{ 1 + \frac{f \dot{e}_0}{[(T_d - f b) + F \operatorname{sgn} \dot{e}_0]} \right\}$$

For step response only, with no torque adaptation we have

$$(14) \quad \sigma = e_0 + \frac{J \dot{e}_0}{f} - \frac{J F}{f^2} \left(\operatorname{sgn} \dot{e}_0 \right) \log \left(1 + \frac{f \dot{e}_0}{F} \right)$$

See illustration on following page, 11a.

Equations (13) and (14) are completely rigorous with respect to the assumptions listed at the beginning of this section; however, we note the sensors of a rotating servomechanism are really measuring the variables of a congruence class, namely

$$\theta_i = \theta + 2\pi n \quad (n = \text{any integer})$$

is a solution of the equations.

Representing $\theta \equiv \theta \bmod 2\pi$ is equivalent to mapping the phase plane on a cylinder. See pages 11b, 11c, 11d.

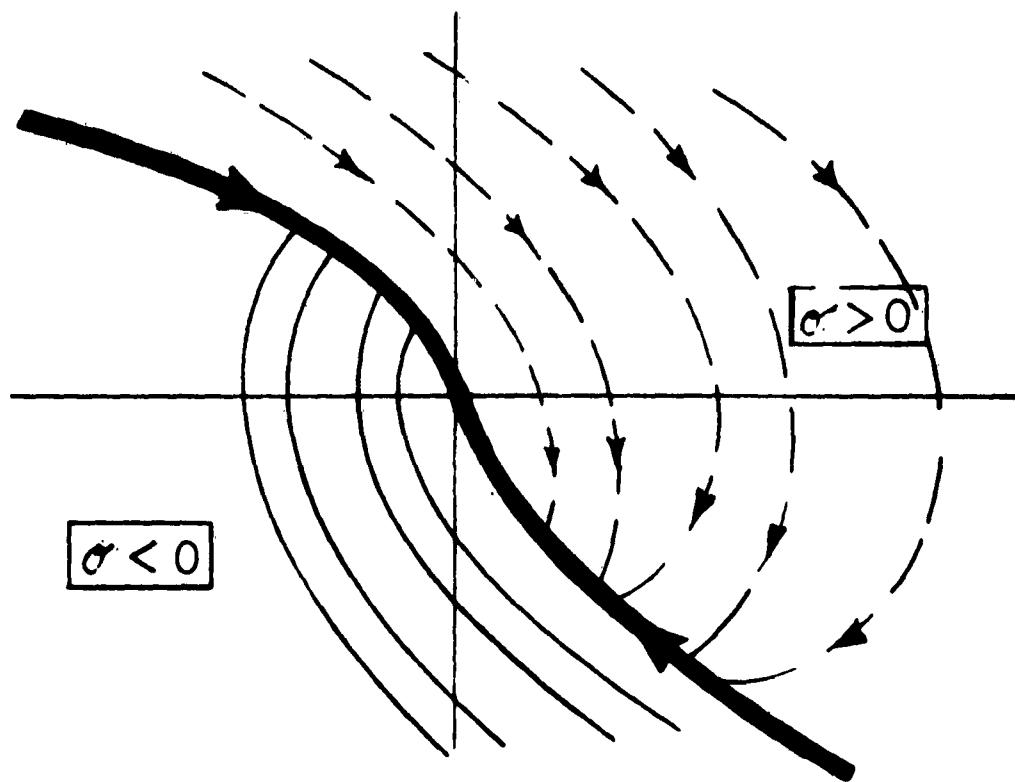


Figure 5.

Phase-plane Trajectories

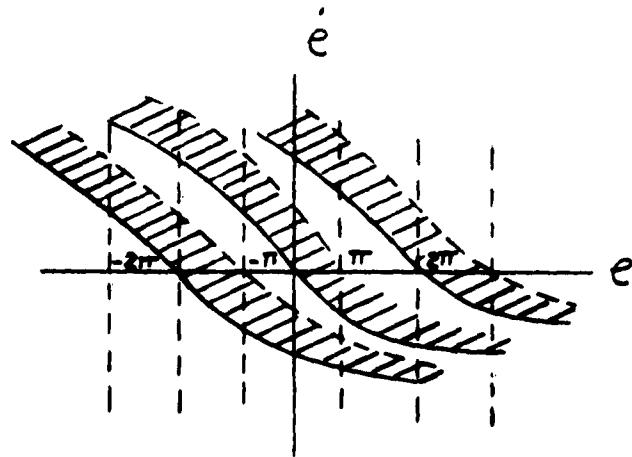


Figure 6.

Cartesian Map of Ambiguous Phase-plane.

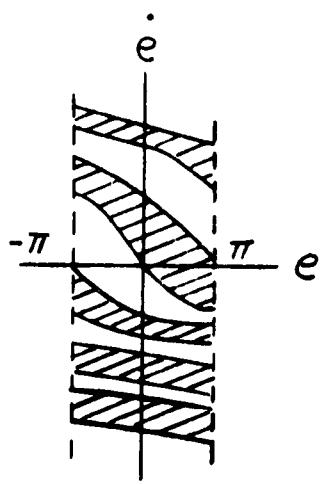


Figure 7.

Truncated Phase-plane Plot

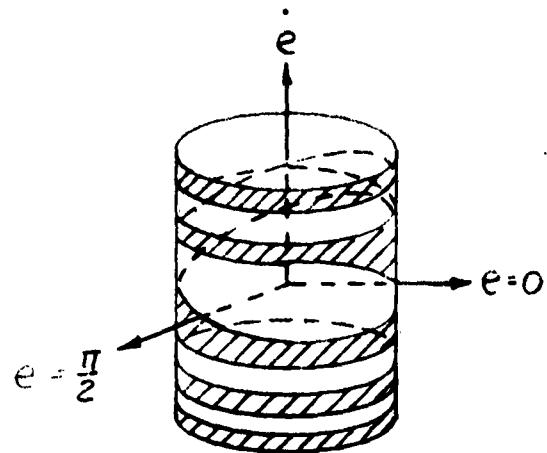


Figure 8.

Cylindrical Map of Phase-plane (ambiguity removed).
(The shaded regions represent $\sigma > 0$)

The construction of a computer analog of this switching criterion is non-trivial. Several structures present themselves, however. Perhaps the most generally useful method is the analog synthesis.

This difficulty with the multipli-connected phase surface is avoided in part by the selection of components.

For example we note that the motor and load are in a closed system in which it is assumed

$$|T_d - f_b| \leq F.$$

Clearly then $|\dot{e}| \leq \frac{2F}{f}$ is the worst case condition. If the e axis is expanded through choice of gear ration then the semi-infinite strip of Figure 9 (page 12a) can be represented in a finite box with as few as two discontinuities in traversing the range of e or \dot{e} . See Figure 10 on page 12b.

Another source of difficulty arises from the fact that no synthesis will be perfectly realized. Analogous to this are the well known oversimplifications of the signum operator, the inherent nonlinearities of the plant, and the inaccuracies of the sensors.

These realities lead one to believe that the system will have several torque transitions in its evolution to the origin. Usually this multiple switching is a limit cycle or a chattering regime or both.

We have, in our analog synthesis program, observed these phenomena in an attempt to put practical limitations on the acceptable tolerances of the servo components. Because our multiplier cabinets have been inoperative since the outset of this program, we have not as yet closed the loop on the analog synthesis of equation (13). We have, however, approximated equation (14) with straight line segments.

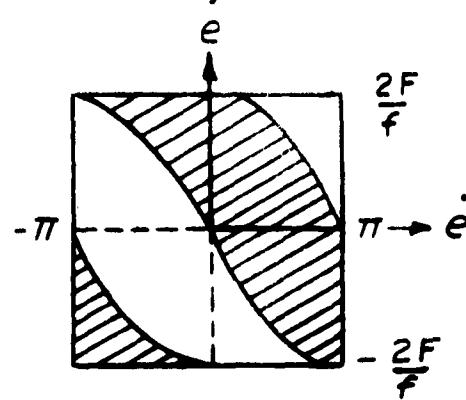


Figure 9.

Cartesian Plot of Phase-plane Showing High Gear Ratio.

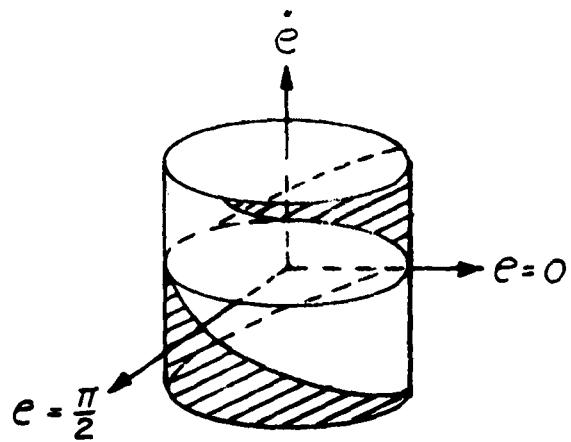


Figure 10.

**Cylindrical Plot of Preceding Page Showing
High Gear Ratio.**

4.1.1 List of Symbols

T = torque

τ_d = disturbance torque at motor shaft

T_d = disturbance torque at load

T_m = developed torque

T_L = load torque

J_L = inertia of load

J_m = inertia of armature

J = equivalent inertia at load

f_L = load friction

f_m = motor friction

f = equivalent friction at load

n = gear ratio

K_e = motor torque constant

K_n = motor damping constant

θ' = shaft displacement

θ = load displacement

E_{in} = terminal voltage

σ = switching function (or control function)

$F = \frac{K_e E_{in}}{n}$

$\theta_i(t)$ = input command

$e(t)$ = error

a = initial displacement of input command

b = rate of change of $\theta_i(t)$

e_o = initial error

$e_{o'}$ = initial error rate

A, B = undetermined coefficients of differential equations

e = base of natural logarithms

t = time

$\text{sgn } \sigma = \text{ signum operator} = 0 \text{ if } (\sigma = 0); = \pm 1 \text{ if } (\sigma \neq 0)$

5.0 CONCLUSIONS AND PROGRAM FOR NEXT INTERVAL

5.1 Practical Implementation

The realization of equation (13) is most easily visualized as a real time analog synthesis. Fast log amplifiers or Schmitt triggers are required. Our first attack was to build up the switching function by this technique. It is now felt that a faster than real time network analog of the linear plant with a sampling circuit and comparator to close the loop will be both economically and technically better. This device will be described in Section 5.3.

Exhaustive catalog searches have led us to the Diehl Mfg. Corp. products as the most useful source of medium power servo components. We have in fact ordered:

- a. Diehl No. FPF85-18-1 Servo-motor
- b. Diehl No. FPF85-29-1 Servo-motor w/Tach
- c. Diehl No. FDE-15-1 D.C. Tachometer

External torque is to be provided by a small hysteresis synchronous motor. For reasons of economy, the inertial load is to be placed at the motor shaft with a precision anti-backlash geartrain (Sterling T628, 299.7:1) driving the control transformer.

In order to maintain a flexible dual mode boundary it was deemed advisable to construct a full power saturating amplifier. This item is in design at present and will be completed by the motor delivery date, approximately 1 December 1961.

5.2 Problem Areas

- a. Sensor measures $\sin \epsilon$, not ϵ , this is demonstrably stable, but leads to non-optimality for steps larger than $\pi/2$ radians.

b. Sensor output is 60 cycle subcarrier, i.e., there exists an inherent 16 millisec time delay between samples.

c. Component drift and long term accuracy prohibits use of practical multipliers and log amplifiers in computation of real time switching functions.

d. Motor controller should have peak power capability of 600 watts to effect smooth torque transition across dual mode boundary. Solid state switching imposes a severe restriction at this power level.

e. Noise in tachometer, in input signals and computer cannot be compensated for by any known technique other than construction of a wide dual mode boundary. These effects in a nonlinear discontinuous controller have not be mathematically investigated and this is beyond the scope of the present project.

5.3 Proposed Solutions

a. The use of induction potentiometers would cure this difficulty, however, noise in the difference amplifier would introduce as many problems. This solution increases the cost of the system without improving performance noticeably. It has been observed that the choice of gear train can all but eliminate this ambiguity. In any event the problem arises only for large values of ϵ and causes only one ambiguous switching even then.

b. At runaway velocity, a 16 millisecond sampling delay could cause a 1.2° error in the output position of the device. Clearly this would not be a steady state error but would be a contribution to the expected transient. Smoothing and linear extrapolation can be accomplished cheaply and should go a long way toward a cure.

c. Analog techniques are avoided in the following scheme:

Construct a network analog which operates faster than real time. This network which will involve active devices, is linear and simulates the physical plant. It will be shown rigorously in the next report that if the real time state variables are applied to the simulator as initial conditions then an a priori selection of the signor will enable determination of the actual signor as soon as the simulated error rate makes its zero crossing. This determination can be done with diode logic.

It appears to be possible to achieve good accuracy at sampling rates in excess of 400 pps. A detailed description will be provided of the entire simulator-computer.

d. Only rarely will the full reverse power of 600 watts be required. A compromise measure has been implemented. A saturating 250 watt transistorized controller is under construction and the dual mode region will be adapted to this.

e. The effects of noise will not be studied until our computer error analysis is instituted.

5.4 Present Status

a. Mathematical and computational work is 60% complete. The remainder will be devoted to computer synthesis.

b. Design of electronic components is 20% complete.

c. Mechanical design, block synthesis and component selection is 90% complete

The remaining 40% of the mathematical and computational work will be in the area of error analysis. A computer routine

has been obtained which will enable us to construct the topology of the dual mode region in order to prevent extraneous switching of the servomechanism in the presence of random additive noise and errors in the calculation of the true switching curve. Computational work needs to be done in the calculation of the linear feedback function to be used within the dual mode region. This work is essentially state-of-the-art control systems engineering and can be done relatively quickly.

Those equipments which have already been designed are the error sampling device, the linear portions of the switching computer and part of the high power control amplifier. Design work remains to be done on the external torque drive source, the non-linear portions of the motor control computer and the high power stage of the linear amplifier.

The remaining mechanical design that is to be completed is the establishment of error criterion and tolerances for the mechanical components associated with the gear train and external torque drive motor.

Test and evaluation will be performed, beginning as soon as the device has been fabricated. It is expected that two man-months of work will be involved.

5.5 Summary

In summary, we feel that great progress has been made in establishing a synthesis technique which will be useful for engineers. The mathematical work has uncovered new but apparently soluble problems connected with the multipli-connected phase-space.

The engineering work has disclosed a new but relatively simple mechanization of the switching function using sampled data techniques. Several practical problems have appeared in the project and have been isolated and either minimized or solved. The final device when delivered will represent a new step forward in the art of relay actuated control systems as well as pointing out several new mathematical difficulties and therefore should be a significant development in control system technology.

6.0 IDENTIFICATION OF PERSONNEL

Dr. R.W. Bass, Chief Scientist

Mr. Wm S. Hare, Manager, Guidance and Control Design Section

Mr. David Maxwell, Senior Engineer

Mr. J.E. Heller, Technician

Resumes of the above personnel are enclosed.

Robert W. Bass

EDUCATION:

Ph.D., Mathematics, Johns Hopkins University
M.A., Mathematics, Oxford University (Rhodes Scholar, Maryland and Wadham)
B.A., Physics, Johns Hopkins University

EXPERIENCE:

1960 - Date Aeronca Manufacturing Corporation, Aerospace Division

1959 - 1960 RIAS (Research Institute for Advanced Study, a Division of Martin Co.). Member, Mathematics Center.

1957 - 1959 1st. Lt., U.S. Air Force. Stationed at Princeton University.

1957 - 1959 RIAS, Inc. Consultant.

1955 - 1956 Princeton University. Research Assistant.

1953 - 1955 Johns Hopkins University. National Science Foundation Fellow and Research Assistant.

MAJOR PROJECTS:

AEC - Project Matterhorn
USAF - Non-linear Mechanics Research Contract
USN - Non-linear Mechanics Research Contract
USA - "Extension of Frequency Method of Analyzing Relay Operated Servomechanisms", a 356-page Research Contract Report to Army Ordnance.

PUBLICATIONS:

"On the Regular Solutions at a Point of Singularity of a System of Non-linear Differential Equations", American Journal of Mathematics, vol. 77 (1955), pp. 734-742.

"On Non-linear Repulsive Forces". Contributions to Non-linear Oscillations, vol. IV (1958), pp. 201-211.

"A Generalization of the Functional Relation $Y(t+s) = Y(t)Y(s)$ to Piecewise-Linear Difference-Differential Equations", Quarterly of Applied Mathematics, vol. 24 (1957), pp. 415-417.

"Improved On-Off Missile Stabilization", Jet Propulsion, vol. 26 (1956), pp. 644-649.

"Improved On-Off Stabilization Systems", Aero Digest, vol. 73 (1956), pp. 28-32.

"Equivalent Linearization, Non-linear Circuit Synthesis and the Stabilization and Optimization of Control Systems", Proceedings, Symposium on Nonlinear Circuit Analysis, Brooklyn Polytechnic Institute MRI Symposia Series, vol. VI (1957), pp. 163-198.

"Remark on Cosmological Models" (with L. Witten), Reviews of Modern Physics, vol. 29 (1957), pp. 452-453.

"Remarks on a Paper of A. M. Letov," Proceedings, Conference on Control Technique, Heidelberg (1956), pp. 209-210.

"The Global Structure of Arbitrary Dynamical Systems", Proceedings of International Congress of Mathematicians, Edinburgh, 1958.

"Zubov's Stability Criterion", Boletin de la Sociedad Matematica Mexicana, vol. 4 (1959), pp. 22-29.

"Alternatives to the Routh-Hurwitz Stability Criterion", Proceedings of the American Mathematical Society (to appear).

"Mathematical Legitimacy of Equivalent Linearization by Describing Functions", Proceedings, International Federation of Automatic Control Congress, Moscow (1960), Butterworth's Scientific Publications, London, pp. 2074-2084.

"Extensions of the Lur'e-Letov-Yakubovich Stability Criteria for Nonlinear Dynamical Systems", Aeronca Technical Report 60-3.

"Reactor Dynamics and Nonlinear Control System Stability Theory", Aeronca Technical Report 60-4.

"A Variational Principle, Determining All Periodic, Autosynthetic Almost-Periodic, and Fixed-Boundary-Configuration Solutions of the N-Body Problem", Aeronca Technical Report 60-5.

"A Generalization to n Dimensions of Bendixson's Non-Existence Criterion", Aeronca Technical Report 60-6.

"A Generalization of D.C. Lewis's Inequalities for Estimating Asymptotic Behavior of Non-Stationary Dynamical Systems", Aeronca Technical Report 60-15.

"Topological Dynamics and Its Applications", Aeronca Technical Report 60-20.

"Functional Analysis and the Global Perturbation of Periodic Solutions", Aeronca Technical Report 60-21.

"Optimal Feedback Control System Design by the Adjoint System", Aeronca Technical Report 60-22.

"The Optimal Space Vehicle Attitude-Control System for Arbitrary Angular Variations, including Nonlinear Coupling" (with P. A. Castruccio and D. L. Slotnick), Aeronca Technical Report 60-23.

"Feedback Computer for Optimal Thrust-Vector Attitude Control and Propulsion Switching in Mid-Course and Landing Maneuvers"*. Aeronca Technical Report 60-24.

"New Methods in Celestial Ballistics"*, Aeronca Technical Report 60-25.

BOOK:

Mathematical Foundations of Nonlinear and Discontinuous Control System Theory (to appear).

SOCIETIES:

Fellow of British Interplanetary Society
American Mathematical Society
American Astronautical Society
American Rocket Society
Society Industrial and Applied Mathematics

AWARDS:

Phi Beta Kappa
Rhodes Scholarship
National Science Foundation Fellowships (both Pre- and Post-Doctoral awards)
Maryland Academy of Science award, "Outstanding Young Scientist of 1960"

*Written jointly with P. A. Castruccio and D. L. Slotnick

WILLIAM S. HARE

EDUCATION:	MS	Electrical Engineering, Johns Hopkins University, Baltimore.
	B.E.S	Electrical Engineering, Johns Hopkins University, Baltimore.
EXPERIENCE:	1961	Aeronca Manufacturing Corporation, Aerospace Division.
	1957 - 1961	Johns Hopkins University. Research Staff and Instructor, Electrical Engineering Department.
	1959 - 1950	Bendix Radio Division, Bendix Aviation Corporation.
	1959	Dowon Engineering Company. Consultant Project Engineer.
	1955 - 1957	Hazeltine Electronics Corporation. Engineer.
	1954	C & P Telephone Company. Engineer.

MAJOR PROJECTS:

U.S.N.	SEADAR System Study
U.S.A.F	E.C.C.M. Research Contract
A.E.C.	Random Number Generator (Biophysics Research Contract with McCollum Pratt Institute)

PUBLICATIONS:

"The Direct Generation of Random Sampling Digits for Digital and Analog Computers". Tech. Report for A.E.C Contract AT (30-1)-1822.

"Memory Devices". ATEE - IRE Prize Paper 1955. The Vector, 6-55.

"Beam Stabilization, Part I & II". Bendix Radio Tech. Report

"Random Search Patterns". Bendix Radio Tech Report

"A Systems Approach to Radar Parameters". Bendix Radio Tech. Report

"Transient Analysis of a Pulse Power Supply" B. Radio Tech. Report

"Private Line Mobile System". A.T. & T. Co. Operating Manual (1955)

SOCIETIES:

Society of Sigma Xi
Eta Kappa Nu
Inst. of Radio Engineers
Amer. Inst. of Elec. Engineers

AWARDS:

Engineering Scholarships, 1951 - 1955
Johns Hopkins University, Gilman
Fellowship, 1957 - 1959.

DAVID J. MAXWELL

EDUCATION: B.E.E Cornell University
Presently working on dissertation toward final requirement for Doctor of Engineering at Johns Hopkins University

EXPERIENCE: 1961 - Date Aeronca Manufacturing Corporation Aerospace Division
1955 - 1961 Bendix Aviation
Bendix Radio Division
(1/2 time employment)

MAJOR PROJECTS: USAF - Electronic steerable array radar
USAF - Antijamming Systems study
COMM - Transistorized IF Amp

PUBLICATIONS: Bendix Radio Technical Reports:
1. A serrodyned traveling wave tube amplifier
2. A wide-band transistorized IF amplifier
3. Effects of limiting before beam-forming on multiple target reception on ESAR
4. A scheme for CW anti-jamming
5. A discrete linear FM pulse compression scheme.

SOCIETIES: Institute of Radio Engineers
ETA KAPPA NU
Sigma XI

JAMES E. HELLER

EDUCATION:

Commercial Radio Institute - Electronics
McCoy College (evening) 2½ years E.E.
Drexel Institute of Technology 3 years - Physics
Presently at McCoy working on B S in physics.

EXPERIENCE:

Nov. 1961 - date Aeronca Mfg. Corporation
Aerospace Division

April 1961 - Nov. 1961 - Martin Co.

June 1959 - April 1961 - Martin/Drexel Co-op student

Sept. 1955 - Sept. 1958 - Martin Co.

MAJOR PROJECTS:

US Army - Lacrosse Missile Power System
USN - P6M GSE
USAF - Titan Trainers

PUBLICATIONS:

None

SOCIETIES: ARS
AIP

<p>Aeronca Mfg. Corp., Aerospace Division, Baltimore, Md.</p> <p>FEASIBILITY STUDY AND EXPERIMENTAL MODEL OF AN OPTIMAL CONTROL SERVOMECHANISM, by Wm. S. Hare, 15 September 1961. 31 p. incl. illus. (SC 87256, DA 36-039) Unclassified report.</p> <p>Using results obtained by R.W. Bass, it has been possible to mathematically describe a phase plane portrait of the switching characteristic of an extreme torque, time optimal, positional-servomechanism which is self adaptive to torque load variations. The approach has been twofold: theoretical work which removes the original simplifications, introduces uncertain parameters and considers the inherent difficulties of extremal systems as immediate problem objectives; practical implementation as the second major effort, has been devoted to considerations of the factors influencing the choice of design, generation of system block diagrams (both functional and operational) and the construction of feasibility hardware to justify our choice of system elements.</p>	<p>UNCLASSIFIED</p> <p>1. Single axis control system I. Hare, Wm. S.</p>	<p>Aeronca Mfg. Corp., Aerospace Division, Baltimore, Md.</p> <p>FEASIBILITY STUDY AND EXPERIMENTAL MODEL OF AN OPTIMAL CONTROL SERVOMECHANISM, by Wm. S. Hare, 15 September 1961. 31 p. incl. illus. (SC 87256, DA 36-039) Unclassified report.</p> <p>Using results obtained by R.W. Bass, it has been possible to mathematically describe a phase plane portrait of the switching characteristic of an extreme torque, time optimal, positional-servomechanism which is self adaptive to torque load variations. The approach has been twofold: theoretical work which removes the original simplifications, introduces uncertain parameters and considers the inherent difficulties of extremal systems as immediate problem objectives; practical implementation as the second major effort, has been devoted to considerations of the factors influencing the choice of design, generation of system block diagrams (both functional and operational) and the construction of feasibility hardware to justify our choice of system elements.</p>	<p>UNCLASSIFIED</p> <p>1. Single axis control system I. Hare, Wm. S.</p>
<p>Aeronca Mfg. Corp., Aerospace Division, Baltimore, Md.</p> <p>FEASIBILITY STUDY AND EXPERIMENTAL MODEL OF AN OPTIMAL CONTROL SERVOMECHANISM, by Wm. S. Hare, 15 September 1961. 31 p. incl. illus. (SC 87256, DA 36-039) Unclassified report.</p> <p>Using results obtained by R.W. Bass, it has been possible to mathematically describe a phase plane portrait of the switching characteristic of an extreme torque, time optimal, positional-servomechanism which is self adaptive to torque load variations. The approach has been twofold: theoretical work which removes the original simplifications, introduces uncertain parameters and considers the inherent difficulties of extremal systems as immediate problem objectives; practical implementation as the second major effort, has been devoted to considerations of the factors influencing the choice of design, generation of system block diagrams (both functional and operational) and the construction of feasibility hardware to justify our choice of system elements.</p>	<p>UNCLASSIFIED</p> <p>1. Single axis control system I. Hare, Wm. S.</p>	<p>Aeronca Mfg. Corp., Aerospace Division, Baltimore, Md.</p> <p>FEASIBILITY STUDY AND EXPERIMENTAL MODEL OF AN OPTIMAL CONTROL SERVOMECHANISM, by Wm. S. Hare, 15 September 1961. 31 p. incl. illus. (SC 87256, DA 36-039) Unclassified report.</p> <p>Using results obtained by R.W. Bass, it has been possible to mathematically describe a phase plane portrait of the switching characteristic of an extreme torque, time optimal, positional-servomechanism which is self adaptive to torque load variations. The approach has been twofold: theoretical work which removes the original simplifications, introduces uncertain parameters and considers the inherent difficulties of extremal systems as immediate problem objectives; practical implementation as the second major effort, has been devoted to considerations of the factors influencing the choice of design, generation of system block diagrams (both functional and operational) and the construction of feasibility hardware to justify our choice of system elements.</p>	<p>UNCLASSIFIED</p> <p>1. Single axis control system I. Hare, Wm. S.</p>

<p>Aeronca Mfg. Corp., Aerospace Division, Baltimore, Md.</p> <p>FEASIBILITY STUDY AND EXPERIMENTAL MODEL OF AN OPTIMAL CONTROL SERVOMECHANISM, by Wm. S. Hare, 15 September 1961. 31 p. incl. illus. (SC 87256, DA 36-039) Unclassified report.</p> <p>Using results obtained by R. W. Bass, it has been possible to mathematically describe a phase plane portrait of the switching characteristic of an extremal torque, time optimal, positional-servomechanism which is self adaptive to torque load variations. The approach has been twofold: theoretical work which removes the original simplifications, introduces uncertain parameters and considers the inherent difficulties of extremal systems as immediate problem objectives; practical implementation as the second major effort, has been devoted to considerations of the factors influencing the choice of design, generation of system block diagrams (both functional and operational), and the construction of feasibility hardware to justify our choice of system elements.</p>	<p>UNCLASSIFIED</p> <p>1. Single axis control system</p> <p>I. Hare, Wm. S.</p>	<p>Aeronca Mfg. Corp., Aerospace Division, Baltimore, Md.</p> <p>FEASIBILITY STUDY AND EXPERIMENTAL MODEL OF AN OPTIMAL CONTROL SERVOMECHANISM, by Wm. S. Hare, 15 September 1961. 31 p. incl. illus. (SC 87256, DA 36-039) Unclassified report.</p> <p>Using results obtained by R. W. Bass, it has been possible to mathematically describe a phase plane portrait of the switching characteristic of an extremal torque, time optimal, positional-servomechanism which is self adaptive to torque load variations. The approach has been twofold: theoretical work which removes the original simplifications, introduces uncertain parameters and considers the inherent difficulties of extremal systems as immediate problem objectives; practical implementation as the second major effort, has been devoted to considerations of the factors influencing the choice of design, generation of system block diagrams (both functional and operational), and the construction of feasibility hardware to justify our choice of system elements.</p>
<p>Aeronca Mfg. Corp., Aerospace Division, Baltimore, Md.</p> <p>FEASIBILITY STUDY AND EXPERIMENTAL MODEL OF AN OPTIMAL CONTROL SERVOMECHANISM, by Wm. S. Hare, 15 September 1961. 31 p. incl. illus. (SC 87256, DA 36-039) Unclassified report.</p> <p>Using results obtained by R. W. Bass, it has been possible to mathematically describe a phase plane portrait of the switching characteristic of an extremal torque, time optimal, positional-servomechanism which is self adaptive to torque load variations. The approach has been twofold: theoretical work which removes the original simplifications, introduces uncertain parameters and considers the inherent difficulties of extremal systems as immediate problem objectives; practical implementation as the second major effort, has been devoted to considerations of the factors influencing the choice of design, generation of system block diagrams (both functional and operational), and the construction of feasibility hardware to justify our choice of system elements.</p>	<p>UNCLASSIFIED</p> <p>1. Single axis control system</p> <p>I. Hare, Wm. S.</p>	<p>Aeronca Mfg. Corp., Aerospace Division, Baltimore, Md.</p> <p>FEASIBILITY STUDY AND EXPERIMENTAL MODEL OF AN OPTIMAL CONTROL SERVOMECHANISM, by Wm. S. Hare, 15 September 1961. 31 p. incl. illus. (SC 87256, DA 36-039) Unclassified report.</p> <p>Using results obtained by R. W. Bass, it has been possible to mathematically describe a phase plane portrait of the switching characteristic of an extremal torque, time optimal, positional-servomechanism which is self adaptive to torque load variations. The approach has been twofold: theoretical work which removes the original simplifications, introduces uncertain parameters and considers the inherent difficulties of extremal systems as immediate problem objectives; practical implementation as the second major effort, has been devoted to considerations of the factors influencing the choice of design, generation of system block diagrams (both functional and operational), and the construction of feasibility hardware to justify our choice of system elements.</p>
		<p>UNCLASSIFIED</p> <p>1. Single axis control system</p> <p>I. Hare, Wm. S.</p>

OASD (R&E), Rm 3E1065 The Pentagon Washington 25, D. C. ATTN: Technical Library	1
Chief of Research and Development OCS, Dept of the Army Washington 25, D. C.	1
Chief Signal Officer Department of the Army Washington 25, D. C. ATTN: SIGORD	1
Director, U. S. Naval Research Laboratory Washington 25, D. C. ATTN: Code 2027	1
Commanding Officer and Director U. S. Navy Electronics Laboratory San Diego 52, California	1
Commander Wright Air Development Center Wright-Patterson Air Force Base, Ohio ATTN: WCOSI-3	2
Commander Wright Air Development Center Wright-Patterson Air Force Base, Ohio ATTN: Lt. Lawrence Schwartz, ASRMCM-2	1
Commander Wright Air Development Center Wright-Patterson Air Force Base, Ohio ATTN: Mr. Phil. Gregory	1
Air Force Office of Scientific Research Washington 25, D. C. ATTN: Captain Bernard Morgan, SRYP	1
Director Aeroballistic Division NASA, Marshall Space Flight Center Huntsville, Alabama ATTN: Helmut H. Horn	1
Commander, Air Force Cambridge Research Center L. G. Hanscom Field Bedford, Massachusetts ATTN: CROTR	1
Commander, Rome Air Development Center Air Research and Development Command Griffiss Air Force Base, New York ATTN: RCSSID	1

Commanding General
U. S. Army Electronic Proving Ground
Fort Huachuca, Arizona

1

Commander
Armed Services Technical Information Agency
Arlington Hall Station
Arlington 12, Virginia
ATTN: TIPDR

10 (Reports)
25 (Proceedings)

Marine Corps Liaison Officer
SIGRA/SL-LNR
U. S. Army Signal R&D Laboratory
Fort Monmouth, New Jersey

1

Electronic Systems Laboratory
Massachusetts Institute of Technology
Cambridge 39, Mass.
ATTN: Dr. L. A. Gould

1

Commanding Officer
Piccatiny Arsenal
Dover, New Jersey
ATTN: Mr. L. F. Nichols

1

University of California Los Angeles
Los Angeles, California
ATTN: Dr. C. T. Leondes
Dept Electrical Engineering

1

Commanding Officer
U. S. Army Signal Equipment Support Agency
Fort Monmouth, New Jersey
ATTN: SIGFM/ES-ADJ

1

U. S. Continental Army Command Liaison Office
U. S. Army Signal Research & Development Laboratory
Fort Monmouth, New Jersey

3 (Final only)

Corps of Engineers Liaison Office
U. S. Army Signal Research & Development Laboratory
Fort Monmouth, New Jersey

1

U. S. Navy Electronics Liaison Office
U. S. Army Signal Research & Development Laboratory
Fort Monmouth, New Jersey

1

Marine Corps Liaison Office
U. S. Army Signal Research & Development Laboratory
Fort Monmouth, New Jersey

1

Commanding Officer
U. S. Army Signal Research & Development Laboratory
Fort Monmouth, New Jersey
ATTN: Logistics Division - Carl Allen

14

Commanding Officer
U. S. Army Signal Research & Development Laboratory
Fort Monmouth, New Jersey
ATTN: Director of Research/Engineering 1

Commanding Officer
U. S. Army Signal Research & Development Laboratory
Fort Monmouth, New Jersey
ATTN: Technical Documents Center 1

Commanding Officer
U. S. Army Signal Research & Development Laboratory
Fort Monmouth, New Jersey
ATTN: SIGFM/EL-ADJ-GDM 1

Commanding Officer
U. S. Army Signal Research & Development Laboratory
Fort Monmouth, New Jersey
ATTN: Technical Information Division 3
(FOR RETRANSMITTAL TO ACCREDITED BRITISH
AND CANADIAN GOVERNMENT REPRESENTATIVES)

Chief, U. S. Army Security Agency
Arlington Hall Station
Arlington 12, Virginia 2

Deputy President
U. S. Army Security Agency Board
Arlington Hall Station
Arlington 12, Virginia 1

This report distributed by Mechanical Engineering Branch, Engineering Design Division,
Engineering Sciences Department, Fort Monmouth, New Jersey.
Telephone: 59-61664

PLEASE DESTROY EXCESS COPIES